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THE EFFECT OF AN INTERFERING SIGNAL ON THE PERFORMANCE OF NONPARAMETRIC DETECTORS USING SPECTRAL ESTIMATES

by

M. J. Wilmut and R. F. MacKinnon

ABSTRACT

Nonparametric statistics have been previously applied to spectral data in order to detect narrow band signals. Here we show the degradation in performance due to the presence of an interfering signal. A modified decision scheme is proposed to overcome this problem. It is shown that with this new technique the presence of even a strong interfering signal does not seriously affect the ability to detect signals.

INTRODUCTION

We consider the situation in which signals embedded in a time series appear as narrow lines in the frequency domain. Detection is based on the matrix of spectral estimates X_{ij}: i=1,2,...,M; j=1,2,. ..,N. Here time is referenced by the i parameter and frequency by the j parameter. The parameter N is the largest number of frequency estimates for which the spectrum can be said to be flat, while M is the number of spectral estimates per frequency cell to be used in the decision process.

Let R_{ij} be the ordered rank of the X_{ij} for a fixed time interval, that is i fixed, j=1,2,...,N. The null hypothesis is that all N frequencies contain noise only. For the X_{ij} independent identically distributed random variables, the R_{ij} are uniformly distributed over the integers 1,2,...,N. An alternative hypothesis is that a signal occurs at at most two of the frequencies represented. In order to determine the distribution of the ranks in this case, one must assume a specific distribution for the X_{ij}; that is, one must specify the nature of the signal and the noise.

Various authors (1), (2), (3) have considered the simple alternate hypothesis that a signal occurs at one frequency. In the following we analyze the performance of the nonparametric decision scheme, which gave good results for the one signal alternative (3), when two signals are present. It is seen that the probability of detection can be seriously degraded by an interfering signal. Finally a modified detector is proposed and analyzed.

DISTRIBUTION OF THE RANK STATISTICS

Suppose $X_1,\ldots,X_N$ represent a set of N independent variable spectral estimates of which the first N-2 correspond to frequencies without signal components (called noise frequencies), X_{N-1} and X_N correspond to cells contain-

ing signals. (We call cell N-1 the interfering signal frequency and cell N the signal frequency). We examine the case where the N estimates are exponential random variables; the first N-2 estimates have mean ψ, X_{N-1} has mean $\psi(1+\theta_2)$ and X_N has mean $\psi(1+\theta_1)$, where θ_1 and θ_2 are the respective cell signal-to-noise ratios and are hence non-negative.

We wish to find P(k) the probability that signal X_N takes rank k where k=1,2,...,N. This occurs if either

a) the interfering signal estimate and k-2 noise frequency estimates are less than X_N and N-k noise frequency estimates are greater than X_N or

b) k-1 noise estimates are less than X_N with the remaining N-k-1 noise frequencies and interfering signal estimate greater than X_N.

We find the probability of one specific ordering of (a) and (b) and then multiply by the number of ways these can occur. Now since $X_1,\ldots,X_N$ are independent, the probability of one specific ordering for (a) is

$$\int f(x)P\begin{bmatrix}\text{k-2 specified noise esti-}\\ \text{mates and interfering}\\ \text{signal estimate are less}\\ \text{than x}\end{bmatrix} P \begin{bmatrix}\text{remaining N-k noise}\\ \text{estimates are greater}\\ \text{than x}\end{bmatrix} dx$$

where P denotes the probability of an event and f(x) is the density function of X_N. The probability of one specific ordering of event (a) is given by

$$\int_0^\infty \frac{1}{2\psi(1+\theta_1)} \exp\left[-\frac{x}{2\psi(1+\theta_1)}\right]\left[1-\exp\left(-\frac{x}{2\psi}\right)\right]^{k-2}\left[1-\exp\left(-\frac{x}{2\psi(1+\theta_2)}\right)\right]\left[\exp\left(-\frac{x}{2\psi}\right)\right]^{N-k} dx$$

$$= \alpha_1 \int_0^\infty \left[1-\exp(-y)\right]^{k-2}\left[1-\exp(-\alpha_2 y)\right]\exp(-\alpha_1 y)\left[\exp(-y)\right]^{N-k} dy$$

where $\alpha_1=(1+\theta_1)^{-1}$, $\alpha_2=(1+\theta_2)^{-1}$, $y=\frac{x}{2\psi}$

$$=\alpha_1\left[\int_0^\infty\left[1-\exp(-y)\right]^{k-2}\exp(-y(N-k+\alpha_1))\,dy-\int_0^\infty\left[1-\exp(-y)\right]^{k-2}\exp(-y(N-k+\alpha_1+\alpha_2))dy\right]$$

which by(4) page 305 Formula 3.312-1 is

$$=\alpha_1\left[B(N-k+\alpha_1,k-1)-B(N-k+\alpha_1+\alpha_2,\ k-1)\right],$$

or replacing B(x,y) the Beta function with the equivalent Gamma function,

$$=\alpha_1\left[\frac{\Gamma(N-k+\alpha_1)\Gamma(k-1)}{\Gamma(N+\alpha_1-1)}-\frac{\Gamma(N-k+\alpha_1+\alpha_2)\Gamma(k-1)}{\Gamma(N+\alpha_1+\alpha_2-1)}\right]$$

Now this situation can occur in $\binom{N-2}{k-2}$ ways. Hence the probability of event (a) is

$$\frac{\alpha_1\Gamma(N-1)}{\Gamma(N-k+1)}\left[\frac{\Gamma(N-k+\alpha_1)}{\Gamma(N+\alpha_1-1)}-\frac{\Gamma(N-k+\alpha_1+\alpha_2)}{\Gamma(N+\alpha_1+\alpha_2-1)}\right]$$

The probability of any specific ordering of event (b) is given by

$$\int_0^\infty\frac{1}{2\psi(1+\theta_1)}\exp\left(-x/2\psi(1+\theta_1)\right)\left[1-\exp(-x/2\psi)\right]^{k-1}\left[\exp(-x/2\psi)\right]^{N-k-1}\left[\exp(-x/2\psi(1+\theta_2))\right]dx$$

$$=\alpha_1\int_0^\infty\left[1-\exp(-y)\right]^{k-1}\exp(-\alpha_1y)\left[\exp(-y)\right]^{N-k-1}\exp(-\alpha_2y)\,dy$$

where α_1 and α_2 are as defined above and $y=x/2\psi$

$$=\alpha_1\int_0^\infty(1-\exp(-y))^{k-1}\exp(-y(N-k-1+\alpha_1+\alpha_2))\,dy$$

$$=\alpha_1B(k,N+\alpha_1+\alpha_2-k-1)=\frac{\alpha_1\Gamma(k)\Gamma(N+\alpha_1+\alpha_2-k-1)}{\Gamma(N+\alpha_1+\alpha_2-1)}$$

Now this specific event can occur in $\binom{N-2}{k-1}$ ways. Hence the probability of

event (b) is $\dfrac{\alpha_1\Gamma(N-1)}{\Gamma(N-k)}\dfrac{\Gamma(N+\alpha_1+\alpha_2-k-1)}{\Gamma(N+\alpha_1+\alpha_2-1)}$

Thus

$$P(k)=P(k,N,\alpha_1,\alpha_2) = \frac{\alpha_1\Gamma(N-1)}{\Gamma(N-k+1)}\left[\frac{\Gamma(N-k+\alpha_1)}{\Gamma(N+\alpha_1-1)} - (\alpha_1+\alpha_2-1)\frac{\Gamma(N-k+\alpha_1+\alpha_2-1)}{\Gamma(N+\alpha_1+\alpha_2-1)}\right] \quad (1)$$

DETECTOR PERFORMANCE

The decision scheme is based on the sum of M independent random variables representing some function of the rank at each frequency. That is, we decide a signal is present at frequency j depending on the value of

$$T_j = \frac{1}{M}\sum_{i=1}^{M} a(R_{ij})$$

where a(x) is some function of x. The statistics of T_j can be found for any function a(x), for small M we use numerical convolution and for large M the Central Limit Theorem. We study below the function which gave the best results for the one signal alternate hypothesis (3), the Savage statistic. Here

$$a(x) = \sum_{\ell=N-x+1}^{N} \ell^{-1}$$

Computer programmes were written to determine the performance of the Savage statistic for various values of M, N, θ_1 and θ_2. The solid lines in figures 1 to 4 give typical results for the original procedure.

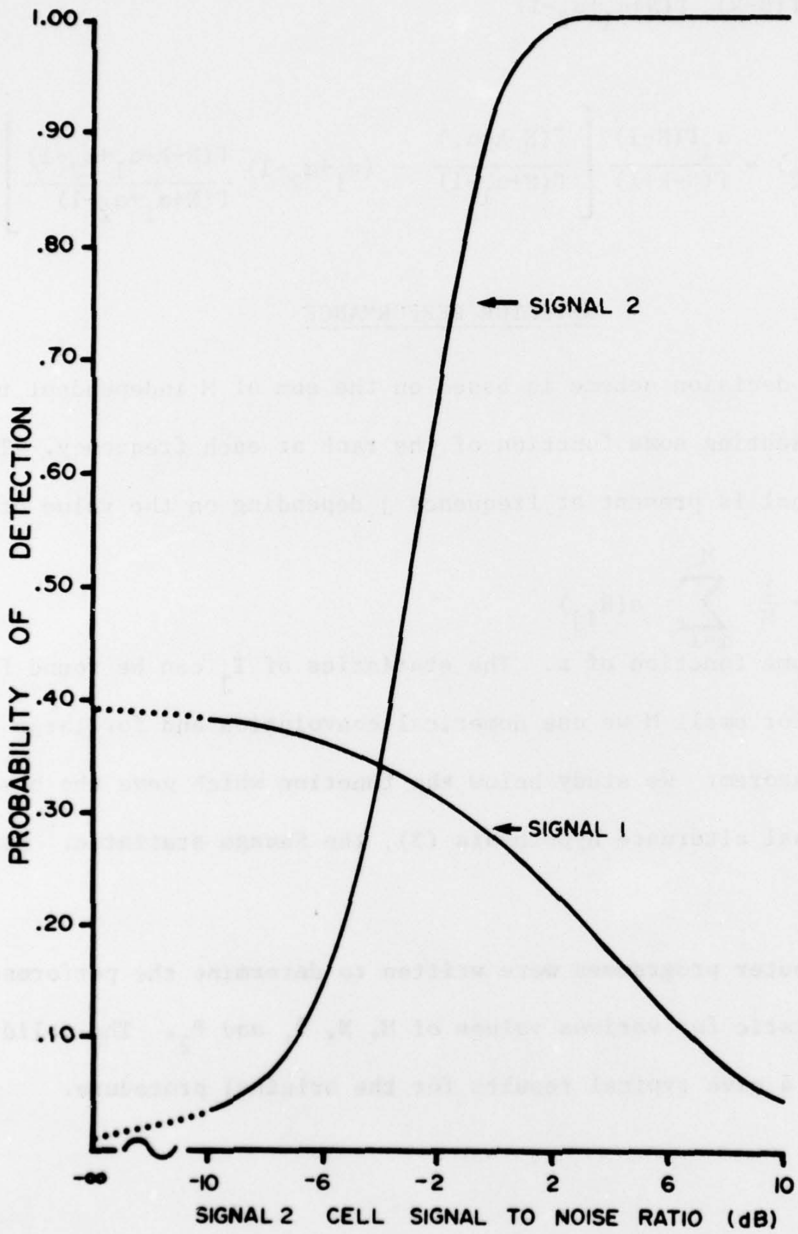

Figure 1. Signal 2 cell signal-to-noise ratio versus probability of detection for signal 1 and for signal 2. False alarm rate 10^{-2}; 40 accumulants, 16 frequencies ranked, signal 1 cell signal-to-noise ratio-4dB.

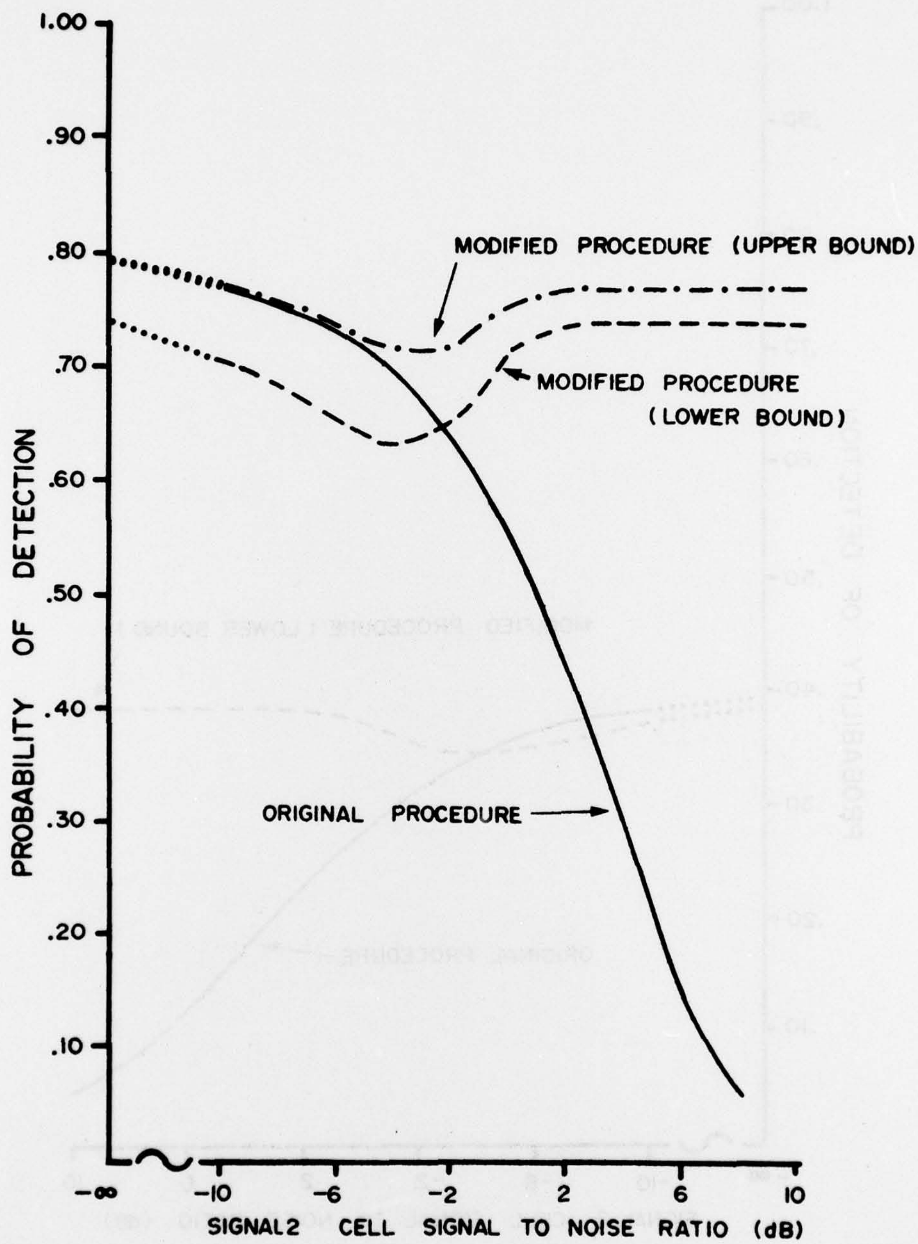

Figure 2. Signal 2 cell signal-to-noise ratio versus probability of detection for signal 1. False alarm rate 10^{-2}; 60 accumulants, 8 frequencies ranked, signal 1 cell signal-to-noise ratio-2dB.

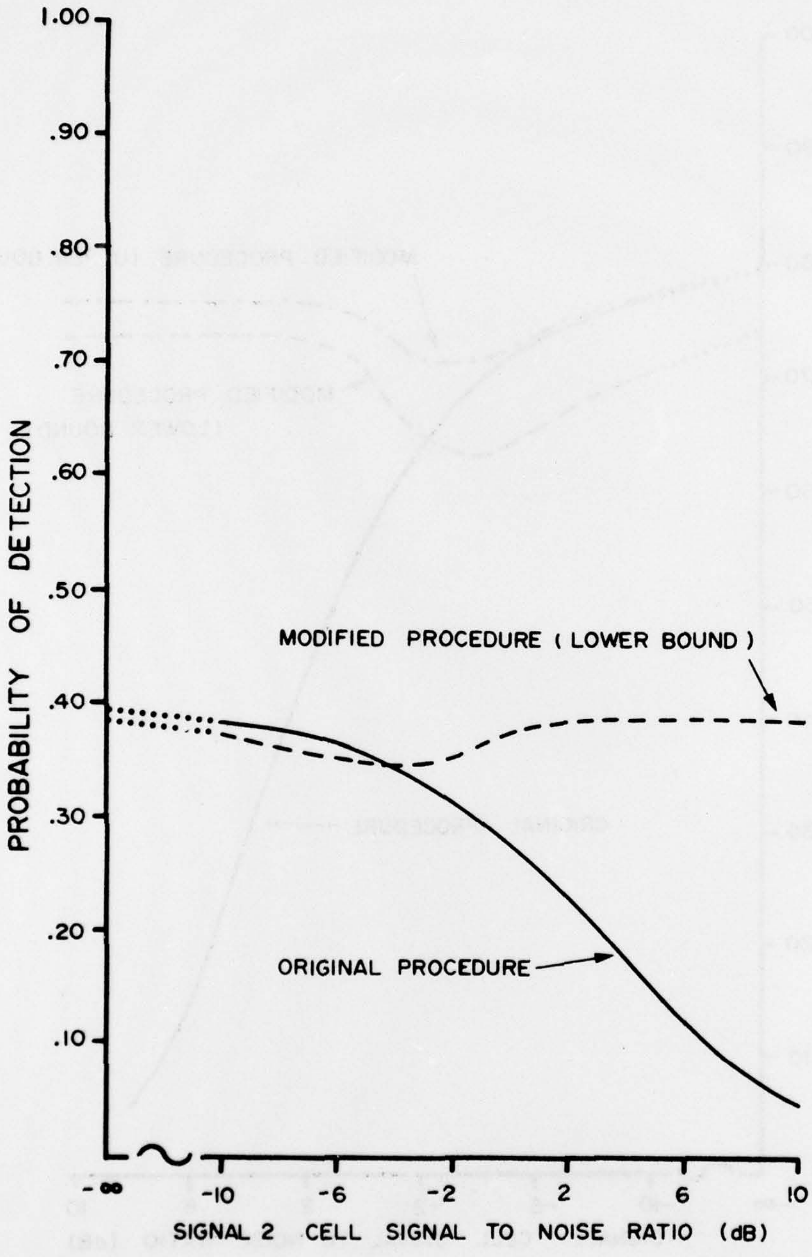

Figure 3. Signal 2 cell signal-to-noise ratio versus probability of detection for signal 1. False alarm rate 10^{-2}; 40 accumulants, 16 frequencies ranked, signal 1 cell signal-to-noise ratio-4dB.

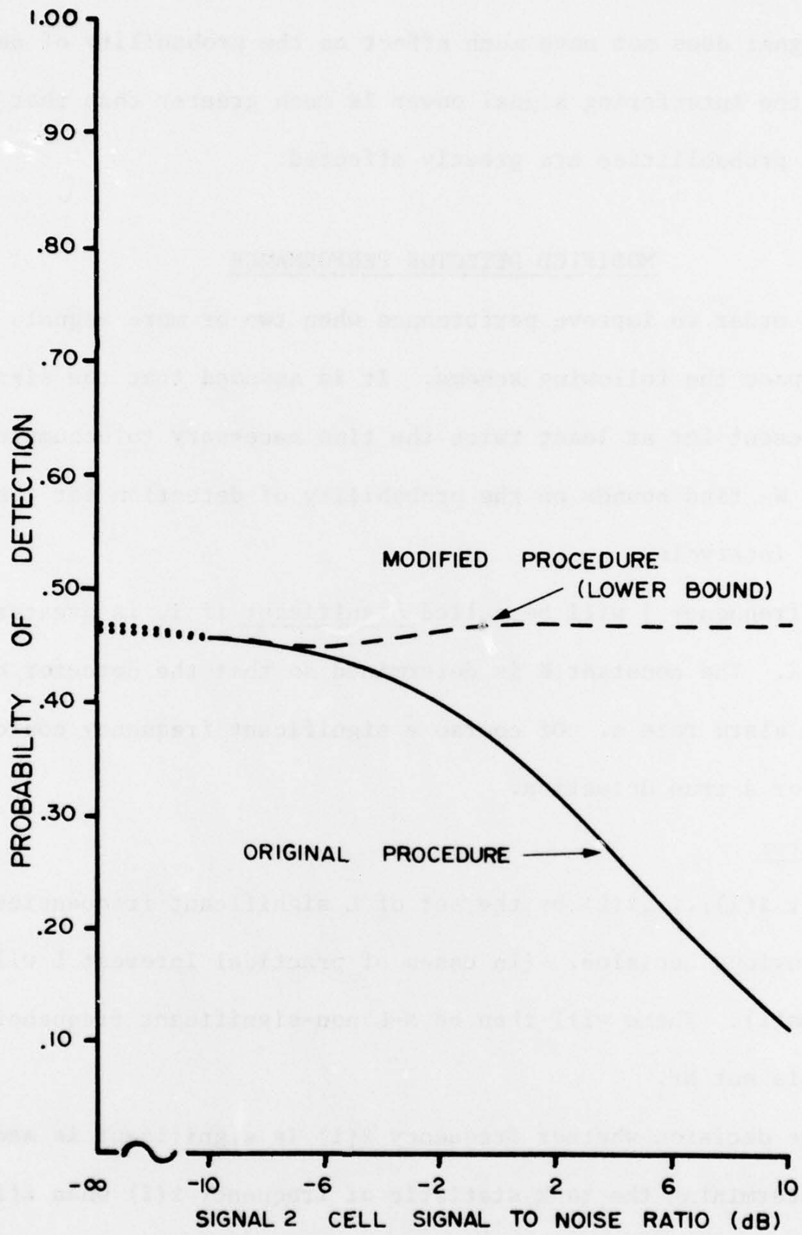

Figure 4. Signal 2 cell signal-to-noise ratio versus probability of detection for signal 1. False alarm rate 10^{-2}; 100 accumulants, 32 frequencies ranked, signal 1 cell signal-to-noise ratio-6dB.

When the interfering signal power is low compared to that of the signal, the interfering signal does not have much effect on the probability of detection. However, when the interfering signal power is much greater than that of the signal, detection probabilities are greatly affected.

MODIFIED DETECTOR PERFORMANCE

In order to improve performance when two or more signals are present we propose the following scheme. It is assumed that the signal of interest is present for at least twice the time necessary to accumulate the statistic T_j. We find bounds on the probability of detection for the second and later time intervals.

A frequency j will be called significant if T_j is greater than some constant K. The constant K is determined so that the detector has a preassigned false alarm rate α. Of course a significant frequency could be either a false alarm or a true detection.

Modified Detector

STEP 1: Let $\ell(1),\ldots,\ell(L)$ be the set of L significant frequencies of the previous decision. (In cases of practical interest L will be small). There will then be N-L non-significant frequencies. Call this set NF.

STEP 2(a): The decision whether frequency $\ell(i)$ is significant is made by determining the rank statistic of frequency $\ell(i)$ when $\ell(i)$ and the N-L frequencies in the set NF are ranked. Suppose L_1 such lines are significant ($L_1 \leq L$).

STEP 2(b) The decision whether a frequency in the set NF is significant is based on the rank statistic of that frequency when the N-L frequencies of that set are ranked. Suppose L_2 such frequencies are

significant ($L_2 \leq N-L$). The false alarm rate is α for all decisions.

STEP 3: Hence a total of L_1+L_2 lines are significant and L_1+L_2 is the value of L for the next decision interval.

In practice we would choose N so that L_1+L_2 is small. The following theoretical analysis keeps the two most significant lines if $L_1+L_2 \geq 3$ and assumes at most two frequencies contain signals.

Detector Performance

Let $P(\underline{\theta_1},\theta_2,N)$ be the probability of detecting a signal of signal-to-noise ratio θ_1 in the presence of an interfering signal of noise power θ_2 when N frequencies are ranked. Similarly $P(\theta_1,\underline{\theta_2},N)$ is the probability of detecting a signal of signal-to-noise ratio θ_2 when we have an interfering signal of signal-to-noise ratio θ_1. We assume some value of M is given.

Let $P(S_1)$ be the overall probability of detecting signal 1. Then $P(S_1) = P(\underline{\theta_1},\theta_2,N)$ for the original procedure.

In order to determine the performance of the modified system we must find the false alarm and detection probabilities under the various hypotheses and compare these results with the original scheme. The comparison is made easier by the fact that for the modified procedure we keep the false alarm rate at a constant α for all decisions. Hence we need only determine the probability of detection for the modified procedure for the alternate hypothesis.

Suppose L=0 and two signals S_1 and S_2 are now present with θ_1 and θ_2 their respective signal-to-noise ratios. For the first decision interval we have

EVENT 1 L=0

$P(S_1|\text{EVENT 1}) = P(\theta_{\underline{1}},\theta_2,N)$

$P(S_2|\text{EVENT 1}) = P(\theta_1,\theta_{\underline{2}},N)$

At the end of this interval L can be 0, 1 or 2. We list the possibilities when L is 1 or 2 and what this means for the following decision interval.

EVENT 2 L=1 and S_1 found previously

$P(S_1|\text{EVENT 2}) = P(\theta_{\underline{1}},\theta_2,N)$

$P(S_2|\text{EVENT 2}) = P(0,\theta_{\underline{2}},N-1)$

EVENT 3 L=1 and S_2 found previously

$P(S_1|\text{EVENT 3}) = P(\theta_{\underline{1}},0,N-1)$

$P(S_2|\text{EVENT 3}) = P(\theta_1,\theta_{\underline{2}},N)$

EVENT 4 L=1 and a false alarm found previously

$P(S_1|\text{EVENT 4}) = P(\theta_{\underline{1}},\theta_2,N-1)$

$P(S_2|\text{EVENT 4}) = P(\theta_1,\theta_{\underline{2}},N-1)$

EVENT 5 L=2 and S_1 and S_2 found previously

$P(S_1|\text{EVENT 5}) = P(\theta_{\underline{1}},0,N-1)$

$P(S_2|\text{EVENT 5}) = P(0,\theta_{\underline{2}},N-1)$

EVENT 6 L=2 and S_1 and a false alarm found previously

$P(S_1|\text{EVENT 6}) = P(\theta_{\underline{1}},\theta_2,N-1)$

$P(S_2|\text{EVENT 6}) = P(0,\theta_{\underline{2}},N-2)$

EVENT 7 L=2 and S_2 and a false alarm found previously

$P(S_1|\text{EVENT 7}) = P(\theta_{\underline{1}},0,N-2)$

$P(S_2|\text{EVENT 7}) = P(\theta_1,\theta_{\underline{2}},N-1)$

EVENT 8 L=2 and two false alarms found previously

$P(S_1|\text{EVENT 8}) = P(\theta_{\underline{1}},\theta_2,N-2)$

$P(S_2|\text{EVENT 8}) = P(\theta_1,\theta_{\underline{2}},N-2)$

The above eight events are mutually exclusive and their union covers the sample space.

For the first interval as L=0 by assumption

$$P(S_1) = P(S_1|\text{EVENT } 1) = P(\underline{\theta}_1,\theta_2,N)$$

For the second interval

$$P(S_1) = P(\text{EVENTS } 3,\ 5 \text{ or } 7 \text{ occur}).\ P(S_1|\text{EVENTS } 3,\ 5 \text{ or } 7 \text{ occur})$$
$$+\ (1-P(\text{EVENTS } 3,\ 5 \text{ or } 7 \text{ occur}))\ P(S_1|\text{NOT EVENTS } 3,\ 5 \text{ or } 7 \text{ occur})$$

Now $P(\text{EVENTS } 3,\ 5 \text{ or } 7 \text{ occur}) = P(S_2 \text{ detected previously})$

$$= P(\theta_1,\underline{\theta}_2,N)$$

Next

$$\text{Min}(P(\underline{\theta}_1,0,N-2),\ P(\underline{\theta}_1,0,N-1)) \leq P(S_1|S_2 \text{ detected previously})$$
$$\leq \max\ (P(\underline{\theta}_1,0,N-2),\ P(\theta_1,0,N-1).$$

In all calculations performed using both the Mann-Whitney and Savage statistics we found $P(\underline{\theta}_1,\theta_2,N-2) \leq P(\underline{\theta}_1,\theta_2,N-1)$. In other words increasing the number of reference noise spectrum estimates ranked increases the probability of detecting signal 1. However this result could not be proved analytically. This result does not hold for all functions a(x).

Thus we may say

$$P(\underline{\theta}_1,0,N-2) \leq P(S_1|S_2 \text{ detected previously}) \leq P(\underline{\theta}_1,0,N-1)$$

$$P(\underline{\theta}_1,\theta_2,N-2) \leq P(S_1|S_2 \text{ not detected previously}) \leq P(\underline{\theta}_1,\theta_2,N)$$

We derive an upper and lower bound on $P(S_1)$ for the second decision interval (assuming the two signals are still present during this time).

$$P(S_1) \geq P(\theta_1,\underline{\theta}_2,N)P(\underline{\theta}_1,0,N-2) + (1-P(\theta_1,\underline{\theta}_2,N))P(\underline{\theta}_1,\theta_2,N-2) \qquad (2a)$$

$$P(S_1) \leq P(\theta_1,\underline{\theta}_2,N)P(\underline{\theta}_1,0,N-1) + (1-P(\theta_1,\underline{\theta}_2,N))P(\underline{\theta}_1,\theta_2,N) \qquad (2b)$$

Similar inequalities hold for $P(S_2)$.

Recursion relations can be derived for $P(S_1)$ and $P(S_2)$ for the third and later intervals. We note some features of these bounds. First if θ_2 is infinite and θ_1 finite, $P(\underline{\theta_1},\underline{\infty},N) = 1$ and we achieve the bounds $P(\underline{\theta_1},0,N-2) \leq P(S_1) \leq P(\underline{\theta_1},0,N-1)$ for the second interval as compared to $P(S_1) = P(\underline{\theta_1},\infty,N)$ for the original procedure. Also it will be seen that for N large the difference between the upper and lower bounds for $P(S_i)$ i=1,2 is almost zero.

Figures 2, 3 and 4 give typical results using the original and modified procedures. Figure 2 shows a case for low N (N=8). We note the upper and lower bounds especially for small values of θ_2 are different.

Figures 3 and 4 illustrate results when N = 16 and 32. Here only the lower bound is given, the upper bound is very close to it.

In all cases there is a degradation in the probability of detection using the modified procedure (as compared to the original procedure) when the interfering signal power is low. This undesirable effect is balanced by the large improvement in detection capabilities when the interfering signal power is large.

CONCLUSIONS

We have determined the effect on a nonparametric detection scheme of an interfering signal. If the strength of this signal is large, signal detection probability can be greatly reduced.

A modified decision procedure was given and lower and upper bounds on its detection capabilities derived. For interfering signals of low strength the modified procedure may be slightly worse than the original whereas for interfering signals of high signal-to-noise ratios the modified procedure gives much better results than the original scheme.

Thus using the modified procedure the presence of a strong interfering signal does not seriously affect the detectability of a weak signal.

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